

Fundamental Analysis of Stocks by Two-stage DEA

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Fundamental analysis of stocks links financial data to firm value in two consecutive steps: a predictive information link tying current financial data to future earnings, and a valuation link tying future earnings to firm value. At each step, a large number of causal factors have to be factored into the evaluation. To effect these calculations, we propose a new two-stage multi-criteria procedure, drawing on the techniques of data envelopment analysis. At each stage, a piecewise linear efficiency frontier is fitted to the observed data. The procedure is illustrated by a numerical example, analyzing some 30 stocks in the Spanish manufacturing industry in the years 1991–1996. Copyright © 2004 John Wiley & Sons, Ltd.

INTRODUCTION

Fundamental analysis of stocks (for references, see next section below) determines the ‘fundamental’ value of a stock by analyzing available information with a special emphasis on accounting information. Over the last decade, accounting researchers have redirected their attention to this task. A number of empirical studies have used information from financial statements to predict future earnings as an indication of the future performance of a firm. Next, the market evaluation of this future earnings-potential is assessed. Comparing with the actual price, the analyst identifies stocks that are overvalued or undervalued. The undervalued ones are candidates for investment and would hopefully earn ‘abnormal’ returns. Most of these studies use econometric techniques to process the information contained in the financial statements. The present paper proposes an alternative methodological approach.

Fundamental analysis can essentially be understood in two different ways. One interpretation is

that fundamental analysis is *predictive*, examining information from financial statements (the financial ‘fundamentals’ of a stock) and generating a forecast of its market value. This is the application, no doubt, that most authors on the subject have had in mind. Another, and quite different interpretation that will occupy us presently, is *normative*. Adopting this perspective, and again inspecting the financial fundamentals of a stock, we shall calculate the market value that the stock ‘should’ (or ‘could’) fetch under some carefully spelled-out circumstances of optimal management and optimal market valuation. For those few corporations that are well managed and well understood by the stock market, this normative value will indeed serve as realistic market forecast. Most corporations, however, will fall short of these idealized circumstances. The normative value will then exceed the actual market performance. To use a term that will be important in the following, the normative value is in the nature of an idealized ‘efficiency frontier’ that a few stocks will attain but most stocks will linger behind.

Fundamental analysis of stocks proceeds in two steps. The first step inspects the financial data of a corporation — its profit-and-loss account and its balance sheet — and aims at assessing future

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earnings. The second step traces the causal link from future earnings to market value. For *both* of these two steps, we shall adopt a normative interpretation. First, we shall calculate an efficiency frontier that traces the idealized relationship between standard financial indicators and revenues. Stocks at the frontier are optimally well managed, converting the various financial inputs into maximal revenues. Stocks falling behind the frontier are less well managed. Second, we shall calculate an efficiency frontier that traces an idealized relationship between various financial data and market value. Stocks at the frontier are optimally priced in the market. Stocks falling behind the frontier are valued less favorably.

To calculate these frontiers numerically, we shall make use of a technique called 'data envelopment analysis' or DEA, for short. It was pioneered by Charnes, Cooper and Rhodes in 1978 (for a recent comprehensive treatment, see Charnes *et al.*, 1994) and fits a piecewise linear envelope or 'frontier' to the given data. The basic idea is easy to explain. Given a collection of points in a multidimensional space, DEA calculates its upper convex hull or 'envelope'. Thus, representing each stock as a point in a multidimensional space, DEA will calculate an envelope frontier to the stocks. The frontier indicates a normative ideal. Stocks located at the frontier are optimally adjusted. Stocks below the frontier are sub-optimally adjusted. For the use of DEA to analyze corporate financial data, see Thore *et al.* (1994), Thore *et al.* (1996) and Thore (1996).

A characteristic feature of fundamental analysis is that it searches for an explanation of stock price and market value via an *un-observed* underlying causal factor: future earnings. Precisely because it is un-observed, fundamental analysis searches deeper, down to the financial fundamentals of the stock. The last step of fundamental analysis (associating future earnings with market value) therefore can never stand on its own. It needs the preceding first step as a prerequisite (associating standard financial indicators of the stock with future earnings).

To represent this cascading causal mechanism mathematically, we propose a novel format of the so-called two-stage DEA. We construct two successive DEA frontiers fitted to the statistical observations, with revenues being an output variable of the first frontier, and an input variable into the second frontier. To be more precise: for

each stock, the idealized and unobserved revenues calculated from the first frontier are fed as inputs into the second frontier.

Following section reviews the fundamental analysis approach to stock valuation. Next section presents the mathematical developments. Following this, we report on the data and on the results of an illustrative numerical example — the two-stage DEA model is estimated for a sample of firms quoted on the Madrid Stock Exchange. Final section sums up.

REVIEW OF FUNDAMENTAL ANALYSIS RESEARCH—STRUCTURING THE FUNDAMENTAL ANALYSIS APPROACH TO STOCK VALUATION AS A TWO-STAGE CAUSAL PROCESS

In the 1970s and 1980s, capital markets accounting research focused on the study of stock market response to the disclosure of accounting information, under the assumption of market efficiency. More recently, some authors have questioned the validity of the market efficiency hypothesis either because it seemed to yield inconclusive results (Lev, 1989) or because anomalies in market behavior were detected (Ball, 1992). Efficiency implies that the market price is a good estimate of intrinsic value. Questioning efficiency, a door is opened to the possibility that the price does not well reflect intrinsic value. In this setting, the objective of fundamental analysis is to determine whether or not current stock prices fully and instantaneously incorporate information about future earnings (or other future economic variables) contained in the fundamental variables (i.e. current prices approximate intrinsic or fundamental value).

Fundamental analysis typically uses econometric techniques like logit/probit analysis (Ou and Penman, 1989; Holthausen and Larcker, 1992; Stober, 1992; Greig, 1992; Bernard *et al.*, 1997; Setiono and Strong, 1998; Charitou and Panagiotides, 1999; Beneish *et al.*, 2001) or regression analysis (Lev and Thiagarajan, 1993; Abarbanell and Bushee, 1997; Sloan, 1996). In order to assess the extent to which stock prices reflect information about future earnings contained in current financial statement data, a test developed by Mishkin (1983) was later applied by

a series of authors (see Sloan (1996), Collins and Hribar (2000), Thomas (2000), Beaver and McNichols (2001), and Xie (2001)). Results from these studies seem to indicate that stock prices do not fully reflect information about future earnings contained in financial information. The conclusion would then follow that the market is inefficient with respect to certain financial statements data.

Assuming that the stock market is not fully able to process the information contained in the financial statements, so that market prices deviate from fundamental values, suitable investment strategies can then be designed. Several authors have indeed claimed that market prices do not instantaneously incorporate all the relevant information contained in the financial statements, and that 'abnormal returns' can be generated (see Ou and Penman (1989), Stober (1992), Holthausen and Larcker (1992), Abarbanell and Bushee (1998), Sloan (1996), Collins and Hribar (2000), Thomas (2000), and Xie (2001)).

Our own approach differs from the econometric estimation employed in all previous studies. We shall use DEA to rank firms on the basis of accounting information. One of the main advantages of this approach is that the valuation exercise is made in a comparative fashion: DEA compares stocks to each other in order to determine their relative efficiency, rather than examining each stock individually. Stocks need to be compared to each other, before the analyst can decide which one offers the best investment opportunities.

Structuring the Fundamental Analysis Approach to Stock Valuation as a Two-Stage Causal Process

According to Ou (1990, p. 145), the observed association between accounting information and stock market value is the result of (i) a link between accounting information and future streams of benefits from equity investments, and (ii) a valuation link between future benefits and stock market values. The disclosure of new accounting information may lead to revisions of investor expectations about future benefits and to corresponding adjustments in current market value.

This implies that the documented association between accounting information and stock prices or stock returns can be understood as the result of a link (the predictive information link) between accounting information and certain value-relevant unobservable attributes.

Although future dividends or future cash flows are usually employed to approximate those unobserved attributes, Ou suggested that future earnings are value-relevant as well — see Ou and Penman (1989), Stober (1992), Setiono and Strong (1998), Charitou and Panagiotides (1999), and others, indicating that investors may have the possibility of using publicly available financial statements information mechanically, applied uniformly across companies, to predict subsequent earnings changes. To sum up the argument, Ou provided evidence that, in terms of financial statements analysis, the relationship between financial data and firm value is established through a two-stage causal process:

- a predictive information link that ties current financial data to projected future earnings, and
- a valuation link that ties projected future earnings to firm value.

Following Ou, then, the purpose of fundamental analysis is to identify hidden or implied causal factors drawn from financial accounting data that can be used to explain the market value of the stock. For our present purposes, we shall assume a chain of causation as follows:

Financial accounting data \Rightarrow Projected earnings
 \Rightarrow Market value.

In this causal process, the factor 'projected earnings' is an intermediary causal factor. It is at the same time the estimated output of the predictive information link (Financial accounting data \rightarrow Projected earnings), and the input into the valuation link (Projected earnings \rightarrow Market value). Thus, the valuation link cannot be established separately, without first estimating Projected earnings.

A couple of elementary accounting relations may be invoked to identify the two links. First, and simplifying, write the Market Value of a corporation as a function of Book Value and Operating Income.

Market Value = $f(\text{Book Value}, \text{Operating Income})$.

Given that Operating Income equals Revenues minus Operating Expenses, this can also be written as

Market Value = $f(\text{Book value}, \text{Revenues}, \text{Operating Expenses})$.

Given this, our aim in the *first stage* will be to use information contained in financial statements ratios in order to project Revenues. Plugging during the *second stage* this projection into the function $f()$ above, together with Operating Expenses and Book Value, the model finally projects Market Value.

As already mentioned, the present work does not deal with the task of predicting market value, nor future earnings. Our concern is normative rather than predictive.

The predictive information link. Our aim is to evaluate the efficiency of management in generating maximal revenues. As recognized by Graham and Dodd (1962), fundamental analysis is a long-term oriented exercise, where the management factor plays an essential role:

Over the long term, forecasting increasingly depends on a correct appraisal of the competence and integrity of management. The company's record demonstrates what ongoing management has accomplished and is the primary source of judgment about the quality of management (ibid., p. 524).

Well-managed companies are more likely to keep generating a steady stream of revenues in the future as well. Hence, there is a link between the past and current record of a company, and its future earnings prospects.

To project Revenues we conventionally assume that the firm aims at maximizing revenues given its available resources. To characterize these inputs and outputs we use information from the balance sheet and from the income statement. Specifically, we have used the following inputs and outputs:

INPUTS:

Accounts receivables
Inventory
Fixed assets
Other assets
Operating expenses

OUTPUT:

Revenues

The inputs account for the economic structure used in the business ('accounts receivables', 'inventory', 'fixed assets' and 'other assets') and for other factors ('operating expenses') that account for the expenses incurred in running the

operations of the firm (i.e. needed for generating revenues).

The valuation link. How does the market translate future earnings into stock value? According to the so-called residual income valuation model (Ohlson, 1995), firm value is expressed as a function of both the book value of equity and the present value of future 'abnormal earnings.' Feltham and Ohlson (1995) showed that operating activities might yield abnormal earnings; hence, an understanding of firm value requires a forecast of future operating profitability (see also Fairfield and Yohn, 2001).

Penman (1998) analyzed how book value and earnings combine to determine stock value. To him, 'future earnings are related to current book values, as well as current earnings, by the intertemporal properties of accounting' (Penman, 1998, p. 294). In this manner, he argued, it would be possible to arrive at a rough determination of the value of a stock without conducting a full pro forma accounting analysis.

In our case, we shall assume that the value of the firm is a function of earnings from operations (revenues and operating expenses) and of the book value of equity:

INPUTS:

Projected Revenues
Operating Expenses
Book Value

OUTPUTS:

Market Capitalization

Projected Revenues and Operating Expenses determine earnings generated during the current period. Earnings not paid to shareholders remain in the firm as retained earnings. The Book Value variable accounts for retained earnings accumulated in the past.

Additionally, one may want to use one or several indicators of price risk as an input at this stage (such as the beta coefficient of the stock).

A CUMULATIVE TWO-STAGE DEA MODEL

For the estimation of the input-output relationships outlined in the preceding section we formulate a two-stage DEA model. For extensive discussions of two-stage DEA, see Charnes *et al.*

(1994) and Sexton and Lewis (2000). Whereas conventional two-stage DEA breaks up into two separate consecutive steps that are estimated separately, our new procedure feeds the projected output of the first step as an input into the second step.

Efficiency in DEA refers to the efficiency (or inefficiency) of a manager to reach the boundary of his *production set* (the set of feasible production points). The production set of the *predictive information link* is simply an extended classical production function, tying sales of a corporation to its inputs like accounts receivables, other assets and operating expenses. The DEA frontier traces the geometrical locus of all Pareto-optimal points of the production set. The piece-wise linear frontier is said to be 'spanned' by its corner points, each such corner point representing an observed corporation that is rated as efficient. Those corporations exhibit 'best practice' in the industry—the management of those corporations that are able to convert the given inputs into the desired outputs more efficiently. Corporations falling behind the frontier are less efficiently operated. The DEA efficiency rating for the predictive information link thus provides a numerical measure of the aptitude of the management team.

The production set of the *valuation link* has to be understood in the attenuated sense of a general input–output relationship, not necessarily dealing with physical production. Such similes are standard in the DEA literature. In the present case, it relates the Projected Revenues, Operating Expenses and the Book Value of equity to the Market Capitalization of the firm stock. Again, the frontier will represent 'best practice' in the industry — the management of those corporations that are able to translate a set of given financial data into the greatest values in the stock market.

For this link as well, efficiency may be tied to management aptitude. Some CEOs are more able to time the product development and promotion activities of their corporation more cleverly than others, creating favorable press coverage and favorable expectations among the community of investment analysts. (Witness the current dot.com mania lifting the stock prices of IPOS to the skies, sometimes even in the absence of both sales and profits.) Corporations falling behind the frontier are run by less genial managers. However,

inefficiency in the valuation link does not mean that *markets* are inefficient. Our analysis does not violate standard assumptions of the efficiency of financial markets.

The model format developed in the present paper employs two consecutive DEA models, one for the predictive information link and one for the valuation link. The two stages are cumulative in the sense that the outputs from the first link are fed as inputs into the second one. In this manner, the explanation of stock prices is established through a two-stage process where the immediate causal factors explaining stock prices actually are unobserved, but instead calculated from an earlier DEA optimization process.

For both links, we shall use the so-called output-oriented version of DEA. For any given vector of inputs, this version calculates the maximal array of outputs that can be obtained. The purpose of the predictive information link is to project the revenues of the company in absolute amounts, given current accounting information on Accounts receivables, Inventory, Fixed assets, Other assets, and Operating expenses. For the valuation link, given Projected Revenues and given Operating Expenses and Book Value, the model calculates the maximal Market Capitalization.

In both stages, we allow for the possibility of variable returns to scale and, hence, the BCC model of DEA will be used (Banker *et al.*, 1984).

To sum up, the objective of the two-stage DEA model is to determine the Pareto-optimal stock price of each company that would take hold if (i) management were as able as that of the best-managed companies in the industry, and (ii) the market were willing to accord the stock a price as high as that accorded to the highest flyers in the market.

Geometric illustration. We have illustrated the two-stage procedure schematically in Figure 1. The figure features three axes: a first-stage input X , a first-stage output (second-stage input) Y , and a second-stage output Z . Events during the first stage are illustrated in the southeast quadrant of the diagram; events during the second stage are illustrated in the northeast quadrant. We have plotted two observations, A and B.

The first observation is represented as A1 in the first-stage quadrant, and as A2 in the second quadrant. A happens to lie on both the first-stage frontier and the second-stage frontier. (The two frontiers have been drawn with thick lines). The

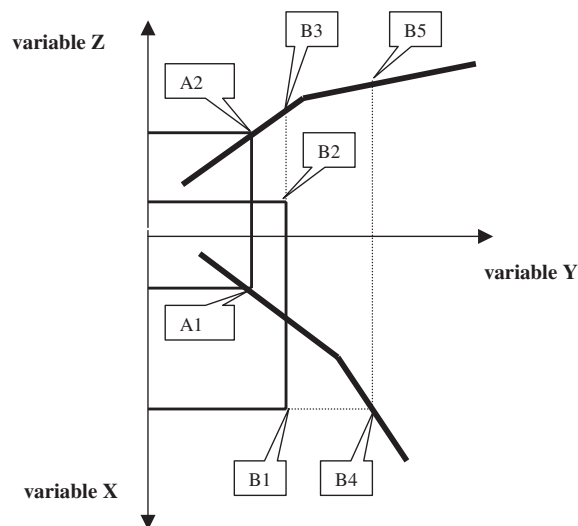


Figure 1. Two-stage DEA procedures illustrated. See the text.

second observation B is represented as B1 in the first-stage quadrant, and as B2 in the second-stage quadrant. This observation is inefficient at both stages.

Following the procedures of conventional two-stage output-oriented DEA, the two observations would be projected on the frontiers as follows. For the first stage, A1 is already located on the frontier and B1 projects onto point B4. For the second stage, A2 is also already located on the frontier while B2 would be projected onto point B3.

Mathematical notation: There are $j = 1, \dots, n$ stocks. For each stock j , the inputs into the first stage X_{ij} , $i = 1, \dots, m$ and the outputs from the first stage Y_{kj} , $k \in K_1 \subset K$ are recorded. (The index k runs over all elements in the set K_1 , which is the set of all outputs from the first stage.)

The inputs into the second stage are written Y_{kj} , $k \in K_2 \subset K$. (This time, the index k runs over the elements of the set K_2 , which is the set of all inputs into the second stage. The sets K_1 and K_2 are not necessarily identical: the set K_1 may contain elements that are not fed into the second stage; similarly, the set K_2 may contain elements that were not brought from the first stage. The set of all outputs from the first stage and all inputs into the second stage is the set K .) The outputs from the second stage are written Z_{rj} , $r = 1, \dots, s$.

For the first stage, consider the output-oriented BCC model

$$\begin{aligned} & \text{Maximize } \psi \\ & \text{subject to } \psi Y_{k0} - \sum_j \lambda_j Y_{kj} \leq 0, \quad k \in K_1, \\ & \quad \quad \quad \sum_j \lambda_j X_{ij} \leq X_{i0}, \quad i = 1, 2, \dots, m, \\ & \quad \quad \quad \sum_j \lambda_j = 1, \\ & \quad \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (1)$$

In the interest of simplicity and expositional clarity, we do not exhibit the slacks in the constraints explicitly. For the numerical calculations reported later, corresponding non-Archimedean formulations were actually employed.

Remember that the fundamentals of the first stage provide a kind of 'hidden' explanation of the stock valuation in the second stage that the second-stage variables alone cannot generate. That is indeed the very essence of the idea of the stock 'fundamentals.' In order to explain the stock valuation it just is not enough to look at the second-stage stock characteristics. One needs to dig deeper in the causal structure, uncovering the underlying first-stage accounting performance.

To accomplish this cumulative explanation from the first stage to the second, we propose a novel feature, *feeding the projected outputs from the first stage as inputs into the second stage*. Using Fig. 1 as an illustration again, we now feed the first-period projected output of the B-observation as the input into the second stage. That is, we feed the input of B4 into the second stage, and the projected result is B5.

Turning to the general case and the mathematical treatment, note that the projected outputs of the first-stage program (1) are

$$Y_{k0}^* = \sum_j \lambda_j^* Y_{kj}, \quad k \in K_1,$$

where the asterisk denotes the optimal solution to program (1). Next, we feed these projected outputs as inputs into the second-stage program (2)

$$\begin{aligned} & \text{Maximize } \psi \\ & \text{subject to } \psi Z_{r0} - \sum_j \lambda_j Z_{rj} \leq 0, \quad r = 1, 2, \dots, s, \\ & \quad \quad \quad \sum_j \lambda_j Y_{kj}^* \leq Y_{k0}^*, \quad k \in K_1 \cap K_2, \\ & \quad \quad \quad \sum_j \lambda_j Y_{kj} \leq Y_{k0}, \quad k \in K_2 - (K_1 \cap K_2), \\ & \quad \quad \quad \sum_j \lambda_j = 1, \\ & \quad \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (2)$$

(The notation $K_1 \cap K_2$ means the intersection of the two sets K_1 and K_2 , that is, the set of all indices k that serve as both outputs from the first stage

and inputs into the second stage. The set $k \in K_2 - (K_1 \cap K_2)$ is the set of all inputs into the second stage that were *not* outputs from the first stage.) The novel feature in program (2) is the set of constraints

$$\sum_j \lambda_j Y_{kj}^* + s_k^- = Y_{k0}^*, \quad k \in K_1 \cap K_2.$$

These constraints feed the projected outputs Y_{kj}^* from stage 1 as inputs into stage 2, rather than the observed inputs Y_{kj} , $k \in K_1$.

DATA DESCRIPTION

The numerical exercises reported in this paper make use of statistical data for Spanish manufacturing firms quoted on the Madrid stock market. The accounting information was taken from the database *Auditorías de Sociedades Emisoras* published by the *Comisión Nacional del Mercado de Valores* (the Spanish Securities and Exchange Commission). It contains the normalized financial statements for companies listed on the Madrid Stock Exchange. The stock market information was extracted from the database *Extel Financial Company Analysis Service*.

Spanish accounting regulations require the parent company of a group to disclose both consolidated financial statement for the group and individual financial statements for the parent as a single firm. The database *Auditorías de Sociedades Emisoras* includes both consolidated and individual information. We decided to focus on the consolidated accounting information, given the existing evidence that it is the consolidated information that is being taken into account when valuing the stocks of parent companies. (Abad *et al.*, 2000 finds evidence that for Spanish firms the consolidated information is more value-relevant than the parent company disclosure alone. Moreover, interviews with Spanish financial analysts reveal that valuations of the parent company are based on group rather than individual accounts, unless the parent company's activities are highly differentiated from the rest of the group's. See Larrán and Rees (1999)).

The variables used as inputs and outputs for both stages were defined as follows:

Inventory = inventories

Accounts Receivables = accounts receivables

Fixed Assets = fixed tangible assets + fixed intangible assets

Other Assets = financial investments + deferred expenses + cash + others

Operating Expenses = cost of goods sold + personnel expenses + depreciation + change in provisions + other operating expenses

Revenues = sales + other operating income

Book Value = capital + retained earnings

Market Capitalization = firms' market capitalization in the Madrid Stock Exchange at the year end.

The DEA calculations require a population of corporations (the DMUs of the analysis) that ideally should be homogenous in terms of common management practices. To obtain a sufficiently great number of observations, we grouped all manufacturing firms together (as opposed to services, utilities and primary products). The number of manufacturing firms in the database *Auditorías de Sociedades Emisoras* was 47 firms in 1991, 48 firms in 1992, 47 firms in 1993, 49 firms in 1994, 49 firms in 1995, and 58 firms in 1996. Dropping firms with lacking information on one or several variables, we ended up with 28 firms in 1991, 29 firms in 1992, 28 firms in 1993, 29 firms in 1994, 29 firms in 1995, and 30 firms in 1996.

NUMERICAL EXERCISE

The two-stage DEA developed in the present paper ranks the performance of each stock relative to each of the two frontiers calculated:

- A first-stage frontier for the predictive information link, indicating the maximal revenues that the company would reach, were its management at par with those of the best-managed companies in the industry;
- A second-stage frontier for the valuation link, indicating the maximal market capitalization at par with the highest flyers in the market.

For each stock, we determined its location relative to the two frontiers. As we shall see, it is possible to identify a group of stocks that consistently stay on the efficiency frontier (in either of the stages) over time.

A novel feature of our two-stage DEA model is the fact that projected or best-practice outputs from the first stage are fed as inputs into the second stage. Actual revenues for all DMUs, and projected or best-practice revenues from the first

stage calculations, are shown in Table 1. Projected revenues and actual revenues are the same for firms that are first-stage efficient. Efficiency ratings for both stages are shown in Table 2.

For first stage inefficient firms, projected revenues are higher than actual revenues, since we are calculating the projected output that the firms could have obtained, provided they used inputs as efficiently as the best managed companies in the industry.

Looking at the results, we note that most of the firms are located at one of the two frontiers at least some of the time (see Table 2). Only five firms (firms 3, 14, 21, 22, and 27) are consistently inefficient year after year.

The success stories. Firms 15 and 16 were efficient at both stages during all six years. In terms of individual outputs, firm 15 had the highest revenues, but firm 16 did not even rank

among the top five firms in terms of revenues. Firm 15 also had the highest market capitalization over the years analyzed; firm 16 ranked in the third place in terms of market capitalization in the first three years and in the second place during the remaining years.

Mixed results. Firms 26, 29 and 30 were efficient in terms of management practice in all six years, but stayed inefficient at the valuation stage. Firms 8 and 23 achieved efficiency at the first stage for most of the years, but never at the second stage. Firm 2 was efficient at the first stage in 1991–1996, and became efficient at the valuation stage in 1996. In most cases, efficiency at the valuation stage went together with efficiency at the first stage.

If a firm is inefficient at any one stage, then the actual market capitalization falls short of the projected capitalization. But if a firm is efficient at both stages, the two concepts coincide. See for

Table 1. Actual and Projected Revenues, in Millions of Pesetas

Firm	1991		1992		1993		1994		1995		1996	
	A.R. ^a	P.R. ^b	A.R.	P.R.	A.R.	P.R.	A.R.	P.R.	A.R.	P.R.	A.R.	P.R.
1	17519	24378	12094	19683	10559	16127	16672	19756	26136	27686	51524	51524
2	40896	40896	37715	37715	35296	35296	29216	29216	24116	24116	2903	2903
3	223251	243518	170722	211499	110944	148656	85503	98236	36951	41968	37616	41858
4	42033	44660	39615	39843	31400	31400	34413	34413	29321	29321	40915	40915
5	10191	10191	8244	8244	7435	7435	5378	6150	48423	48423	44210	44210
6	—	—	—	—	—	—	—	—	—	—	13851	13851
7	15925	15925	16654	17077	15835	15961	17126	17126	17608	17608	19194	19194
8	48950	48950	55928	55928	57770	57770	69660	69660	83621	84924	91314	91314
9	32812	32812	37188	37188	32613	34603	24693	26931	21510	22824	23690	26718
10	45509	45509	43333	43333	46670	46670	55533	58498	117050	121742	103603	103603
11	25437	26009	23473	23473	26017	26017	27746	28994	33888	33942	33896	34308
12	47502	47502	48525	48896	47386	48576	49620	50904	50940	51337	50774	54983
13	28643	30116	38281	39298	35417	36790	40084	40663	41382	41382	—	—
14	75937	81456	65232	67388	63596	67388	68343	70473	75689	82045	15651	19260
15	654315	654315	703969	703969	673174	673174	787677	787677	782801	782801	821608	821608
16	36578	36578	43234	43234	48443	48443	50702	50702	56288	56288	58476	58476
17	19904	23386	15891	19288	18909	21374	29941	30422	33042	33042	31655	34138
18	5065	5065	4505	4505	—	—	—	—	—	—	—	—
19	59068	63429	44719	52379	44559	49887	37982	38539	42199	42199	42939	46389
20	16449	16449	12462	12462	16782	16782	18790	19235	15921	15921	15813	15813
21	17374	17876	17999	19207	24801	27567	30585	32717	38611	45416	41636	48442
22	36533	38082	40321	44166	43681	49991	49019	56008	61348	66678	56818	64553
23	31985	31985	33464	33464	39609	39609	45087	45087	47808	48329	50502	50502
24	—	—	7936	7936	8563	8563	9827	9827	11489	11489	14236	14236
25	5201	5201	8260	8572	4803	4803	5310	5310	6167	6167	6152	6152
26	248394	248394	288325	288325	309603	309603	378969	378969	395073	395073	464902	464902
27	51839	53213	55586	55769	57299	60269	58723	60172	61259	64214	65824	70047
28	—	—	—	—	—	—	29527	29527	31188	31188	34057	34057
29	459011	459011	545145	545145	490315	490315	627722	627722	676472	676472	723839	723839
30	33246	33246	35019	35019	30608	30608	35912	35912	39391	39391	42188	42188
31	127556	127556	146967	149419	143772	144226	157234	157234	157314	157314	163389	163389
32	—	—	—	—	—	—	—	—	—	—	19366	19366

^aA.R. Actual revenue.

^bP.R. Projected revenue.

Table 2. DEA Ratings at Each Stage

Firm	1991		1992		1993		1994		1995		1996	
	1st stage	2nd stage	1st stage	2nd stage	1st stage	2nd stage	1st stage	2nd stage	1st stage	2nd stage	1st stage	2nd stage
1	1.39	6.82	1.63	11.91	1.53	3.02	1.18	2.87	1.06	2.90	1.00	3.95
2	1.00	7.90	1.00	4.79	1.00	16.43	1.00	7.41	1.00	6.82	1.00	1.00
3	1.09	4.17	1.24	10.27	1.34	8.19	1.15	4.15	1.14	5.53	1.11	7.73
4	1.06	1.75	1.01	1.00	1.00	1.33	1.00	1.23	1.00	1.00	1.00	1.20
5	1.00	5.03	1.00	2.90	1.00	1.00	1.14	1.00	1.00	5.41	1.00	3.66
6	—	—	—	—	—	—	—	—	—	—	1.00	2.09
7	1.00	5.43	1.02	3.33	1.01	5.38	1.00	3.62	1.00	4.11	1.00	3.91
8	1.00	1.49	1.00	2.26	1.00	2.58	1.00	2.88	1.02	5.32	1.00	4.05
9	1.00	7.21	1.00	7.90	1.06	14.63	1.09	4.14	1.06	1.00	1.13	2.02
10	1.00	4.31	1.00	3.49	1.00	4.51	1.05	2.44	1.04	2.61	1.00	2.99
11	1.02	3.53	1.00	4.00	1.00	5.23	1.04	4.87	1.01	4.91	1.01	5.08
12	1.00	3.25	1.01	4.22	1.02	4.45	1.03	4.02	1.01	5.51	1.08	7.24
13	1.05	1.36	1.03	1.26	1.04	3.55	1.01	3.28	1.00	3.10	—	—
14	1.07	7.40	1.03	8.28	1.06	4.02	1.03	4.25	1.08	4.96	1.23	1.89
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
17	1.17	6.38	1.21	7.31	1.13	6.79	1.02	5.74	1.00	4.45	1.08	5.22
18	1.00	1.00	1.00	1.00	—	—	—	—	—	—	—	—
19	1.07	12.27	1.17	24.18	1.12	33.33	1.01	1.00	1.00	1.00	1.08	1.56
20	1.00	1.00	1.00	1.82	1.00	2.67	1.02	3.63	1.00	3.46	1.00	6.40
21	1.03	2.12	1.07	2.54	1.11	3.70	1.07	4.69	1.18	8.16	1.16	9.08
22	1.04	1.74	1.09	3.52	1.14	5.12	1.14	4.11	1.09	3.92	1.14	5.04
23	1.00	1.20	1.00	2.02	1.00	1.55	1.00	2.11	1.01	2.96	1.00	2.60
24	—	—	1.00	1.19	1.00	1.58	1.00	1.00	1.00	1.00	1.00	1.05
25	1.00	1.00	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.51
26	1.00	6.01	1.00	4.69	1.00	5.60	1.00	3.63	1.00	4.33	1.00	5.11
27	1.03	3.97	1.01	4.80	1.05	11.55	1.02	5.95	1.05	8.35	1.06	10.05
28	—	—	—	—	—	—	1.00	7.37	1.00	8.52	1.00	12.40
29	1.00	2.97	1.00	2.03	1.00	1.14	1.00	1.25	1.00	3.52	1.00	3.15
30	1.00	2.73	1.00	4.42	1.00	5.91	1.00	6.62	1.00	6.02	1.00	8.82
31	1.00	1.00	1.02	1.48	1.01	1.83	1.00	1.81	1.00	2.03	1.00	1.98
32	—	—	—	—	—	—	—	—	—	—	1.00	1.00

instance Figure 2, illustrating the results for firm 19. This firm was close to first-stage efficiency in the three 1991–1993 years. But the overall result is nevertheless dominated by the poor showing of the firm at the second stage, and the actual capitalization falls far behind the projected one. In 1994 and 1995, firm 19 was efficient at both stages. Only then do the actual and the projected capitalization coincide.

Some additional insight can be obtained by looking at the market-to-book ratios of the consistently efficient and the consistently inefficient companies. Firm 16 all the time hovered between the top three firms in terms of market-to-book ratios and firm 15 between the top eleven firms. Firm 32 entered the stock market in 1996 with a very high market-to-book ratio and reached efficiency in both the first and second stage.

The laggards. The market-to-book ratios for those firms that never reached the best-practice

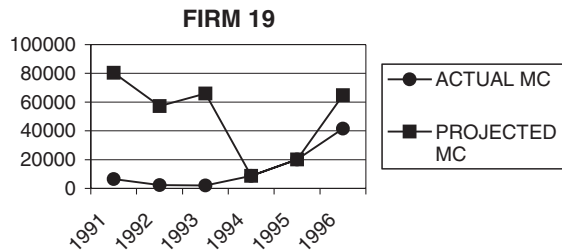


Figure 2. Capitalization results for firm 19.

frontier are less than one. There are only two exceptions to this observation: firm 3 in 1993, 1994 and 1995, and firm 27 in 1991 and 1994. The market value apparently reflects the fact that these companies are far from the best-managed companies in the industry.

Finally, when looking at the ranking of firms in terms of size, it is not possible to infer any clear relationship between size and efficiency; in fact,

some of the largest companies (like firm 3) are inefficient at both stages, and there also are large firms that are efficient at both stages (like firm 15).

CONCLUDING REMARKS

The basic notion of the so-called fundamental analysis in accounting and finance is the idea that the stock-market performance of a corporation can be causally linked to underlying or 'fundamental' financial characteristics to be found in the profit-and-loss account and the balance-sheet. The association is supposed to be established through an intermediary but non-measured variable: expected future earnings. Various financial ratios or underlying financial statistics brought from the books of the corporation are supposed to determine expected earnings. In their turn, expected earnings determine the stock price.

Employing a novel twist to mathematical frontier analysis, we have shown how a two-stage DEA model can be used for the purpose of fundamental analysis. In the first stage, a frontier is estimated that ties current accounting information to the future firm's performance. At the second stage, we calculated an efficiency frontier that traces the idealized relationship between certain accounting information and market value.

The special feature of the two-stage DEA model proposed in this paper is the fact that *projected or best-practice revenues calculated in the first stage* are fed as inputs into the second stage. In this way, information from the first stage calculations is taken into account when running the second stage DEA. The efficiency rating achieved in the second stage is influenced by the firms' relative performance in the first stage. In fundamental analysis it is not enough to look at the firm's earnings figure. It is also necessary to understand how the firm is performing in relation to other firms in the industry and how well it generates earnings.

In our empirical application we employed data brought from manufacturing companies listed on the Madrid Stock Exchange. The results indicate that it is possible to identify groups of companies that consistently stay at one or both of the two efficiency frontiers over several years. We have also been able to spot trends in behavior for some of the firms.

Our reformulation of fundamental analysis throws some sidelight on the issue of the possibility of generating 'abnormal returns' on a stock portfolio. Such returns would accrue on stocks whose fundamental values exceed their market prices. The use of DEA to analyze financial data does not by itself violate the efficient market hypothesis. Nor does it support it. Whether investment in a sub-frontier stock (whose DEA-projected stock price exceeds its market price) will yield abnormal returns or not, one simply does not know.

In our empirical investigation using data from the stock market in Madrid, we did not generate any abnormal returns. This market is fractured and institutionally less well developed than US markets, and current prices may therefore only imperfectly mirror efficiency prices. To test whether abnormal returns are possible in some other institutional setting, fresh investigations are needed.

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